A major purpose of the Technical Information Center is to provide the broadest dissemination possible of information contained in DOE's Research and Development Reports to business, industry, the academic community, and federal, state and local governments.

Although a small portion of this report is not reproducible, it is being made available to expedite the availability of information on the research discussed herein.

1

LA-UR -87-285

CONF 870453--1

TITLE: DETAILED MSW CALCULATIONS FOR SOLAR ^{8}B NEUTRINOS: PROBABILITIES FOR ELECTRON NEUTRINOS TO REMAIN ELECTRON NEUTRINOS AT EARTH AS A FUNCTION OF ENERGY, Δm^2, AND $\sin^2 2\theta$

LA-UR--87-285

AUTHOR(S): S.P. ROSEN and J.M. GELB

DE87 006075

SUBMITTED TO:

To be submitted to Santa Barbara Workshop on Solar Neutrinos, Institute for Theoretical Physics, UCSB, 29 April - 2 May 1987

Los Alamos

Los Alamos National Laboratory
Los Alamos, New Mexico 87545

DETAILED MSW CALCULATIONS FOR SOLAR ^8B NEUTRINOS:

PROBABILITIES FOR ELECTRON NEUTRINOS TO REMAIN ELECTRON NEUTRINOS AT EARTH AS A FUNCTION OF ENERGY, Δm^2, AND $\sin^2 2\theta$

S.P. ROSEN and J.M. GELB

Theoretical Division, Los Alamos National Laboratory
Los Alamos, New Mexico 87545

JANUARY 1987

DETAILED MSW CALCULATIONS FOR SOLAR ^8B NEUTRINOS:

PROBABILITIES FOR ELECTRON NEUTRINOS TO REMAIN ELECTRON NEUTRINOS AT EARTH AS A FUNCTION OF ENERGY, Δm^2, AND $\sin^2 2\theta$.

S.P. ROSEN and J.M. GELB

THEORETICAL DIVISION
LOS ALAMOS NATIONAL LABORATORY
LOS ALAMOS, NEW MEXICO 87545

January 1987

ABSTRACT

We present the results of detailed two-flavor MSW calculations for solar ^8B neutrinos. The probabilities for solar neutrinos to remain electron neutrinos at earth are calculated as functions of energy, Δm^2, and $\sin^2 2\theta$, and the results are presented in both graphical form and numerical form. The numerical results are contained on an IBM PC compatible floppy diskette which also contains the ^8B spectrum. Thus all of the data can be combined to predict results for any detector scheme.

In this report we present the detailed results of solar neutrino oscillation computations in which the effect of matter oscillations as they travel through the sun, the MSW effect, is taken into account. We calculate the probabilities, $P(\nu_e \to \nu_e;\ E)$ for electron neutrinos born in the sun to remain electron neutrinos when they arrive at earth as functions of their energy E and the oscillation parameters Δm^2 and $\sin^2 2\theta$ associated with two-flavor models. We use the solar density profile of the standard model and average the computed probabilities over the production region in the solar core associated with ^8B neutrinos.

The formalism for these calculations and the method of carrying them out are described at length in section V and Appendix A of our original publication[1], which we include as an appendix to this report. Several differences between these computations and our original ones should be noted:

(1) A larger range of energy with finer increments is provided: we cover the range 0.05 to 20.0 MeV in steps of 0.05 MeV instead of 1 to 14 MeV in steps of 1 MeV.

(2) A more general range of parameters (Δm^2 and $\sin^2 2\theta$) not necessarily tied to the ^{37}Cl experiment of Davis et al.[2,1] is provided.

(3) A slightly different electron density profile, provided by J.N. Bahcall[3], was used (see Fig. 1).

(4) We averaged Eq. 5.2 in our original publication[1] over the range of distances of the earth-sun orbit ($R_{es} - \epsilon$ to $R_{es} + \epsilon$; $R_{es} = 1.5\times10^8$ km and $\epsilon = 0.017 \cdot R_{es}$). We find

$$\langle \sin^2(\pi R_{ee}/L) \rangle = \frac{1}{2}\left[1 - \frac{\sin(2\pi\varepsilon/L)}{(2\pi\varepsilon/L)} \cdot \cos(2\pi R_{ee}/L)\right] \tag{1}$$

$$\langle \sin(\pi R_{ee}/L) \cdot \cos(\pi R_{ee}/L) \rangle = \frac{1}{2}\left[1 + \frac{\sin(2\pi\varepsilon/L)}{(2\pi\varepsilon/L)} \cdot \sin(2\pi R_{ee}/L)\right] \tag{2}$$

Figures 2 through 14 show the results of our calculations. Each figure refers to a specific value of Δm^2, beginning with 10^{-4} $(eV)^2$ in figure 2 and decreasing in uniform steps of $10^{-0.5}$ to 10^{-10} $(eV)^2$ in figure 14. The upper three curves in each figure correspond to the small values of $\sin^2 2\theta$ = 0.001, 0.01, and 0.1 and are labeled as such; the lower three curves correspond to the large values $\sin^2 2\theta$ = 0.3, 0.7, and 1.0 and are again labeled appropriately. Precisely the same information contained in the figures is presented in the form of numerical data files on the floppy diskette.

The behaviour of the probability curves can be understood rather well from the general properties of $P(\nu_e \to \nu_e)$ as a function of $E/\Delta m^2$. As can be seen from figures 1, 3, and 4 in our original paper[1] (see the Appendix to this paper), the general shape of $P(\nu_e \to \nu_e)$ is that it remains close to one for $E/\Delta m^2$ less than 10^5, then drops rapidly to a small value and remains small for half a decade or more in $E/\Delta m^2$, depending upon the value of $\sin^2 2\theta$, and finally begins a steady climb back to unity as $E/\Delta m^2$ increases to values of order 10^8 and higher. The earlier part of the curve corresponds to the adiabatic approximation which tends to the "small" assymptotic value of $\sin^2\theta$ as $E/\Delta m^2$ increases; the size of the region in which $P(\nu_e \to \nu_e)$ has this value depends upon the value of $\sin^2 2\theta$ itself, being one decade for $\sin^2 2\theta$ = 0.01 and two decades for $\sin^2 2\theta$ = 0.2 (see figure 1 of the appendix). As the probability begins its climb back from $\sin^2\theta$ to unity, we enter the region of the "sudden", non-adiabatic approximation.

Let us now consider the shapes of the probability curves for small mixing angles as Δm^2 pregresses from 10^{-4} to 10^{-10} $(eV)^2$. From figure 2 we see that for the low energy neutrinos we are still in the adiabatic regime and so the probabilities are large at first, then fall rapidly to very small values as the energy increases, and tend to stay small. Some increase occurs towards the high energy end of the $\sin^2 2\theta = 0.001$ curve because the "suppression gap" is rather small for such a small mixing angle. As Δm^2 decreases, the neutrino energy span (0-20 MeV) moves to the right in the $P(\nu_e \to \nu_e)$ versus $E/\Delta m^2$ diagram and we move steadily away from the adiabatic regime into the non-adiabatic one. Consequently the region of large probabilities disappears as Δm^2 gets smaller, to be replaced by a region of small probabilities and then by the region in which the probability steadily climbs back to unity. This progression occurs most rapidly for very small $\sin^2 2\theta$ ($\simeq 0.001$) and less rapidly for the larger values ($\simeq 0.1$), but it is complete in all cases by the time Δm^2 has fallen to 10^{-8} $(eV)^2$. For smaller values of Δm^2, oscillation lengths become comparable with the earth-sun distance and so we begin to see in vacuo oscillations coming into play; they are particularly visible for $\sin^2 2\theta \sim 0.1$.

Turning to the large mixing angle curves, we see that they essentially begin, for $\Delta m^2 \simeq 10^{-4.5}$ in the "suppression gap" where the probability for $\nu_e \to \nu_e$ equals $\sin^2\theta$; thus the $\sin^2 2\theta = 1.0$ line is higher than that for $\sin^2 2\theta = 0.7$, which in its turn is higher than $\sin^2 2\theta = 0.3$. This behaviour continues until Δm^2 falls to about $10^{-6.5}$, at which point we begin to move into the non-adiabatic regime. Since the "suppression gap" is narrower for smaller $\sin^2 2\theta$, the lowest value shown, namely $\sin^2 2\theta = 0.3$ is the first to begin increasing to unity; it is then followed by the $\sin^2 2\theta = 0.7$ curve, and finally by $\sin^2 2\theta = 1.0$. By the end of this process, between $\Delta m^2 = 10^{-7.5}$ and 10^{-8} $(eV)^2$, the order of probabilities has been inverted, the $\sin^2 2\theta = 0.3$ having the highest probability for $\nu_e \to \nu_e$ at any energy, followed by $\sin^2 2\theta = 0.7$, and then 1. For smaller values of Δm^2, the effects of in vacuo oscillations between the sun and the earth begin to set in; superimposed upon these oscillations is a "beat" phenomenon associated with the factor $\{\sin(2\pi\varepsilon/L) \; / \; (2\pi\varepsilon/L)\}$ in equations (1) and (2), which arises from the average between the apogee and perigee of the orbit of the earth. The mean values of the probabilities are given by the standard formula $P(\nu_e \to \nu_e) = 1 - 1/2 \sin^2 2\theta$.

The usefulness of these graphs is demonstrated in our recent work with J.N. Bahcall on the MSW effect in solar neutrino-electron scattering[4]. We computed products of the from $\sigma(E) F_\nu(E) P(E/\Delta m^2)$ where σ is the cross-section for ν_e-e or ν_μ-e scattering, $F_\nu(E)$ is the flux of solar ^8B neutrinos (see figure 15), and $P(E/\Delta m^2)$ are the probabilities for $\nu_e \to \nu_e$ at earth or 1-P for $\nu_e \to \nu_\mu$ at earth (figures 2 through 14). From these products we can extract the spectrum of electrons as a function of their energies and the total event rates as a function of Δm^2.

IBM FLOPPY DISKETTE: COMPUTER FILE INFORMATION

Upon request, we can send you a copy of a standard 5 1/4" double-sided double-density floppy diskette formatted for IBM personal computers. The files are standard ASCII files and can be used by a personal computer or uploaded to a mainframe.

B8SPEC.DAT is the ^8B solar neutrino spectrum with the range: 0.05 to 20.0 MeV in increments of 0.05 MeV (it contains 400 points). An example Fortran-77 program to read the file is given below:

```
      DIMENSION ENERGY(400),B8(400)
      OPEN(UNIT=1,NAME='B8SPEC.DAT',STATUS='OLD',FORM='FORMATTED')
    5 FORMAT(F7.4)
      DO 10,I=1,400
      ENERGY(I)=FLOAT(I)*0.05
      READ(1,5)B8(I)
   10 CONTINUE
      CLOSE(UNIT=1)
      STOP
      END
```

The probability spectra (Figs. 2 through 14) are contained in files of the form B8##P#.DAT, each containing 2406 points. B810P0.DAT corresponds to $\mathrm{Log}(\Delta m^2) = -10.0$, B807P5.DAT corresponds to $\mathrm{Log}(\Delta m^2) = -7.5$, etc. Within each file there is the first value of $\sin^2 2\theta$ followed by 400 points (0.05 to 20.0 MeV by 0.05 MeV) followed by the next value of $\sin^2 2\theta$ and its spectrum, etc. There are six values of $\sin^2 2\theta$ in the following order: 0.001, 0.01, 0.1, 0.3, 0.7, and 1.0 . Note: 2406=(1+400)*6 .

An example Fortran-77 program to read the files is given below:

```
      DIMENSION SS2T(6),ENERGY(400),PROB(400)
      CHARACTER*15 FILE
    5 FORMAT(F7.4)
      TYPE *,'ENTER FILE TO BE READ'
      ACCEPT *,FILE
      OPEN(UNIT=1,NAME=FILE,STATUS='OLD',FORM='FORMATTED')
      DO 10,I=1,6
      READ(1,5)SS2T(I)
      DO 20,J=1,400
      READ(1,5)PROB(I,J)
      IF(I.EQ.1)ENERGY(J)=FLOAT(J)*0.05
   20 CONTINUE
   10 CONTINUE
      CLOSE(UNIT=1)
      STOP
      END
```

REFERENCES:

(1) S.P. Rosen and J.M. Gelb, Phys. Rev. D 34, 969 (1986). Included as an appendix at the end of this report.

(2) R. Davis et al., in Science Underground, proceedings of The Workshop on Science Underground, Los Alamos, New Mexico, edited by M.M. Nieto et al. (AIP Conf. Proc. No. 96), (AIP, New York, 1983).

(3) J.N. Bahcall et al.,Rev. Mod. Phys. 54, 767 (1982).

(4) J.N. Bahcall, J.M. Gelb, and S.P. Rosen, Los Alamos Preprint LA-UR-86-3802. Submitted to Phys. Rev. D.

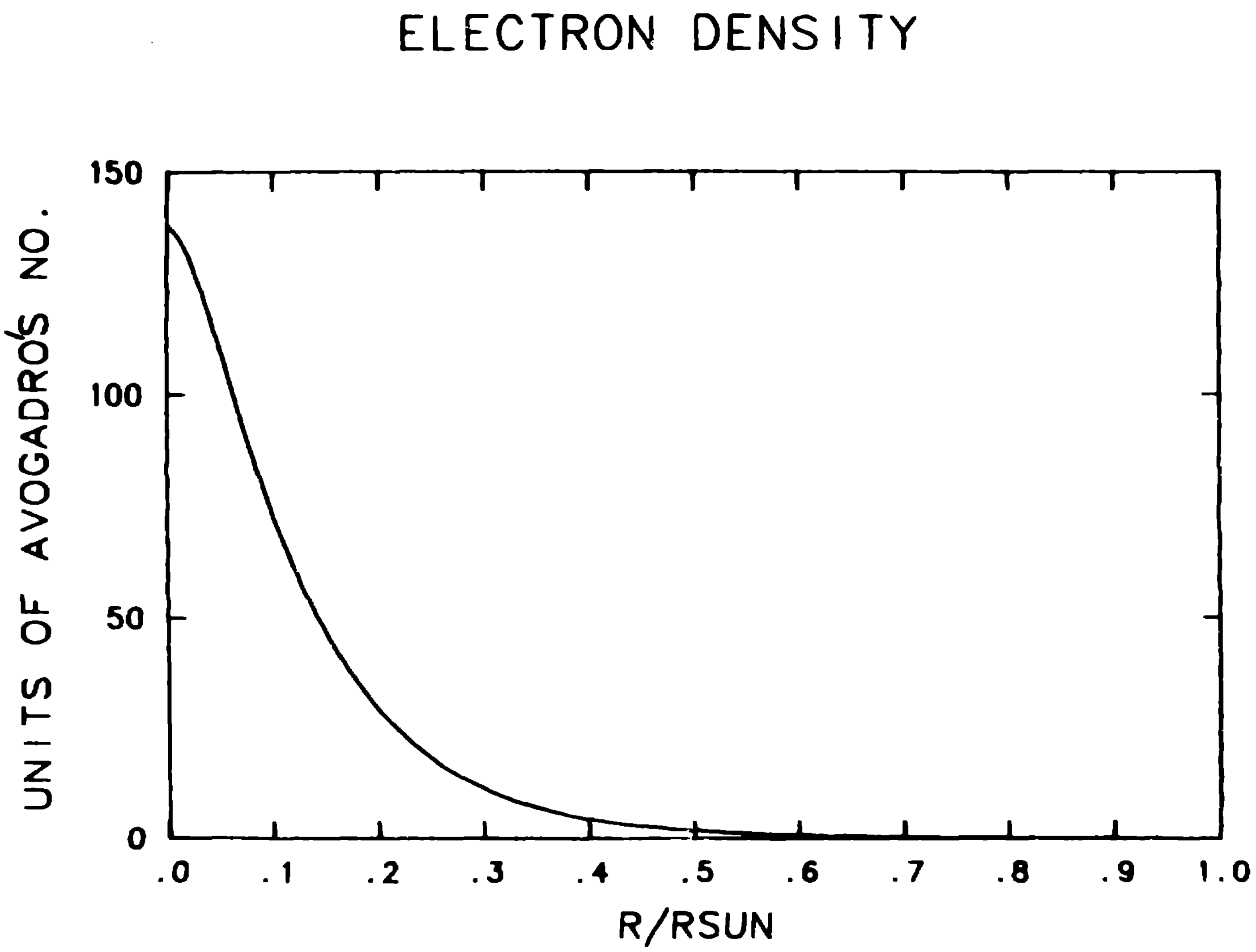

ELECTRON DENSITY
UNITS OF AVOGADRO'S NO.
150
100
50
0
.0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
R/RSUN

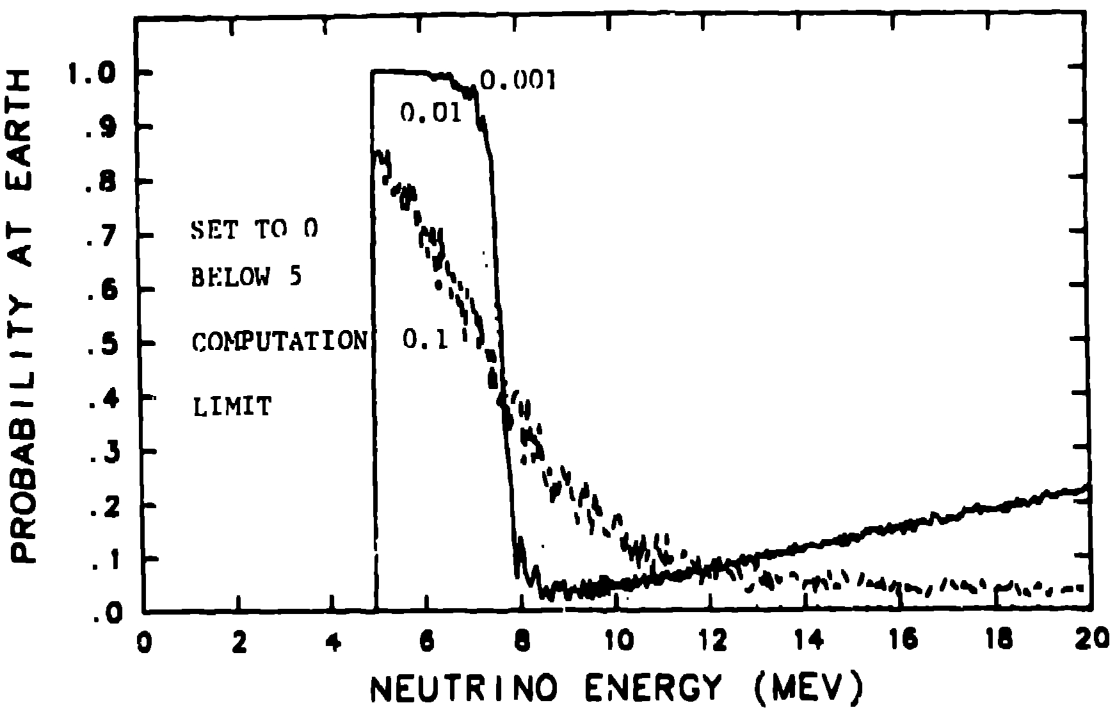
FIGURE 1

NOTES FOR FIGURES 2 THROUGH 5:

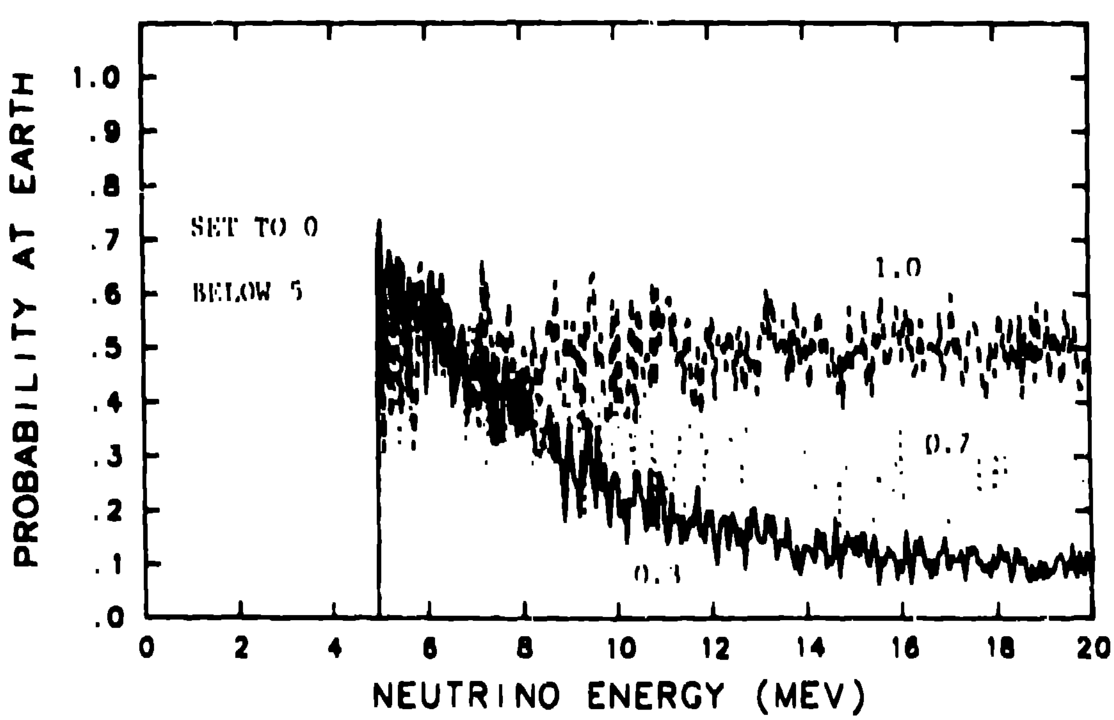
The neutrino oscillation length goes as $E/\Delta m^2$. For small values of $E/\Delta m^2$ greater detail is required as the neutrino propagates through the sun. This becomes exceedingly time consuming for certain values of $E/\Delta m^2$. Therefore we did not compute probabilities below 5 MeV for $\mathrm{Log}(\Delta m^2) = -4.0$ through -5.5. For smaller values of Δm^2, $E/\Delta m^2$ is large enough so that computations below 5 MeV were carried out. Most ^8B detection schemes "look" at energies higher than 5 MeV anyway; therefore, we simply set the probabilies in Figs. 2 through 5 equal to zero below 5 MeV so that the computer files are of the same format for all values of Δm^2. In the event that we decide to calculate the rest of the probabilities we will make updates of the graphs and computer files. If you find that you need these probabilities at present, then you can try to interpolate the data based on the graphs and the theories of the MSW effect and add the new data to the files.

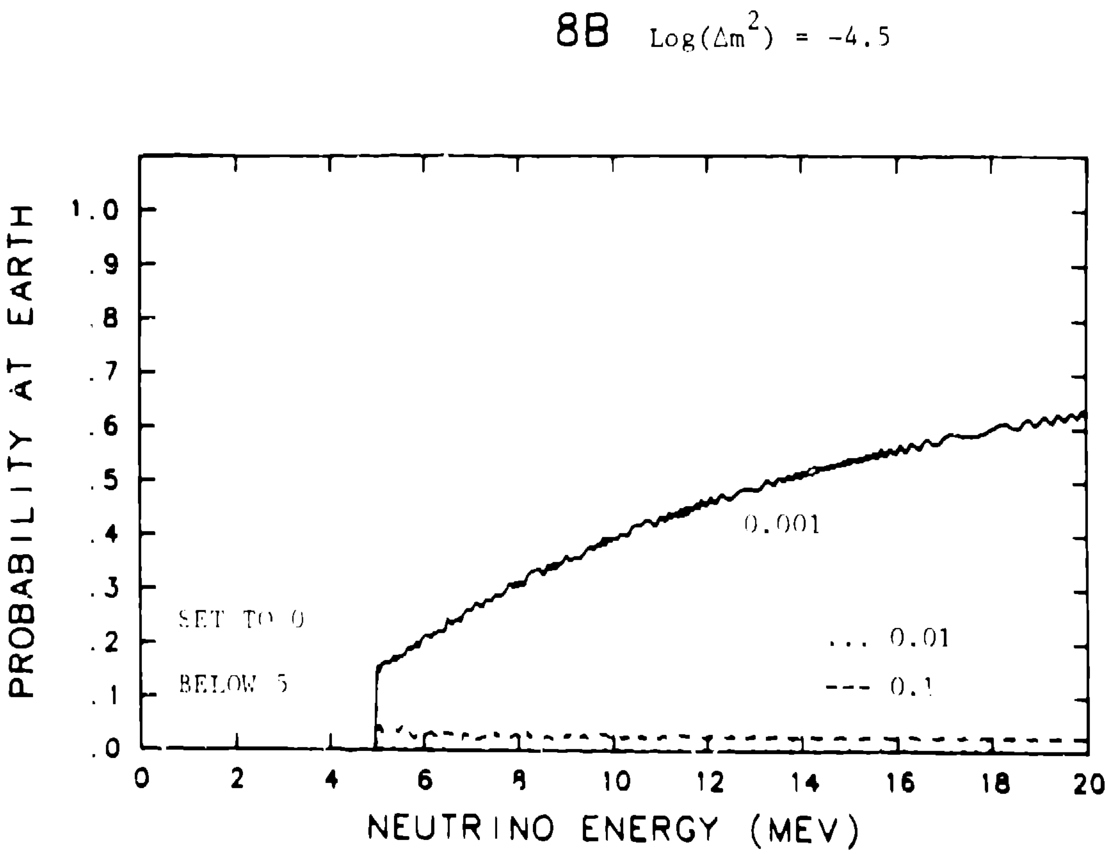
FIGURE 2

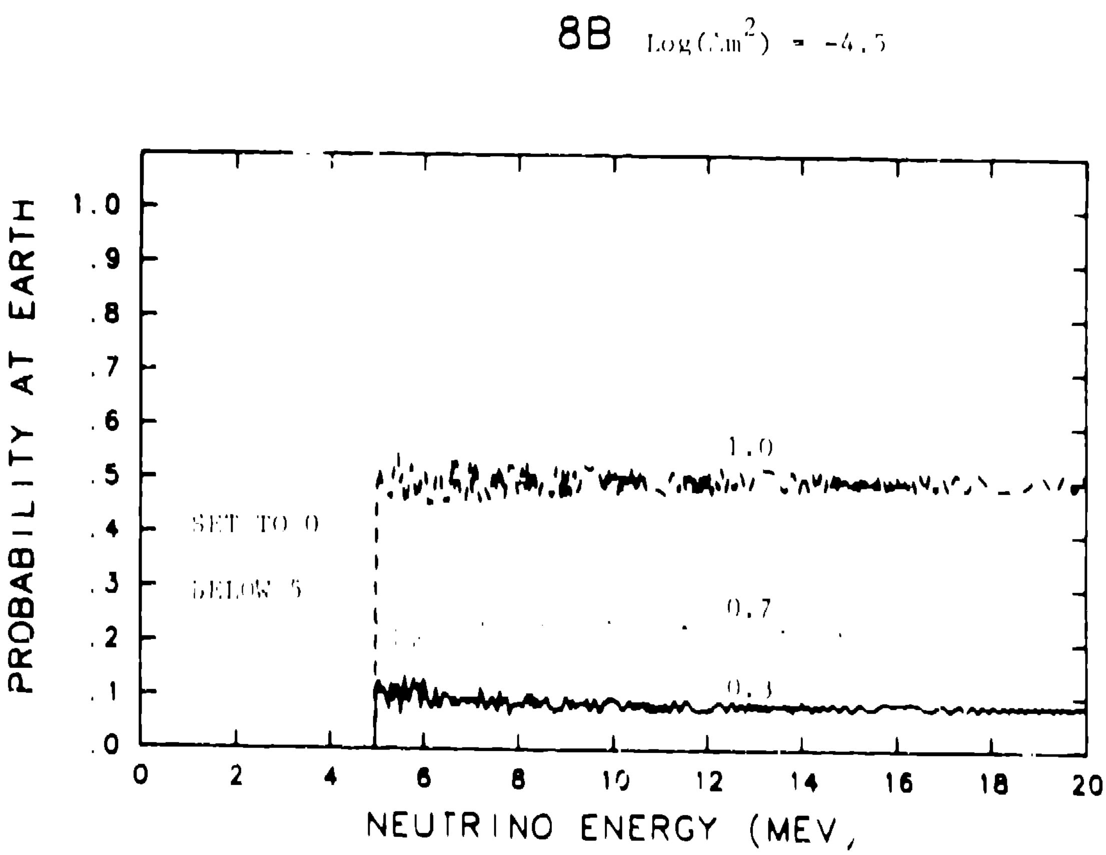
8B $Log(\Delta m^2) = -4.0$

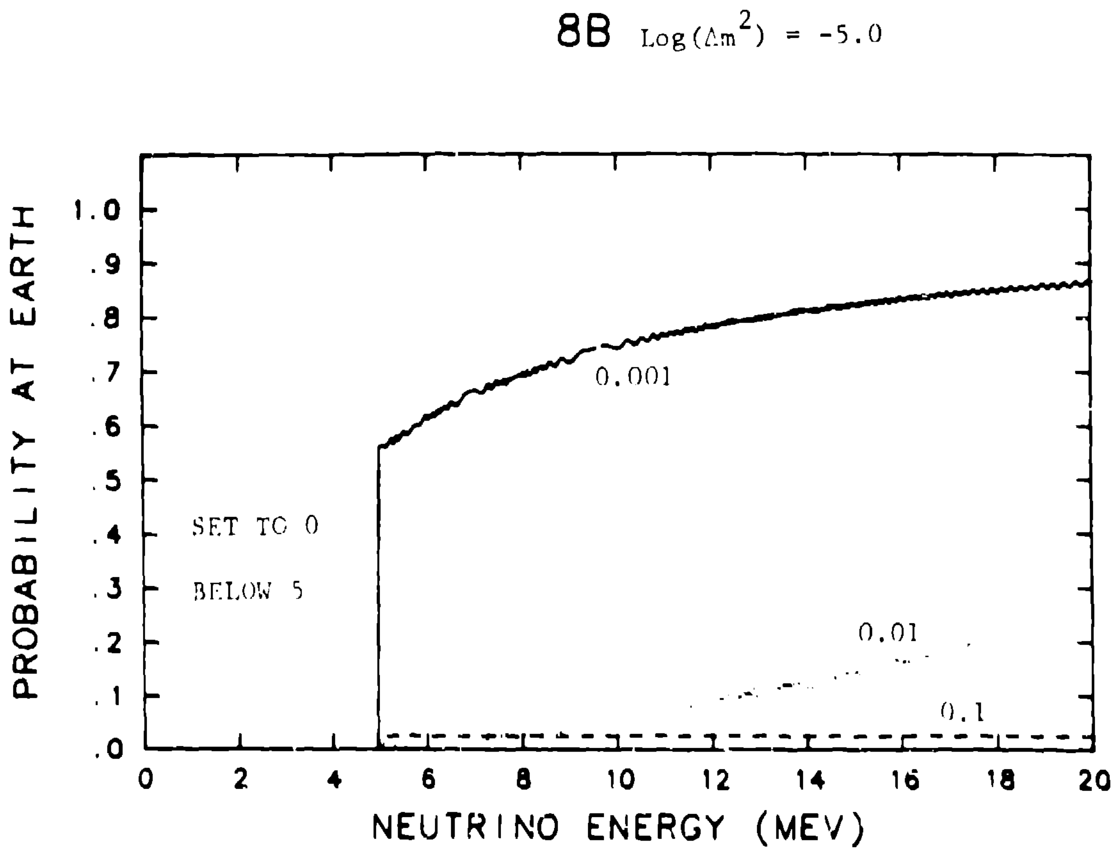


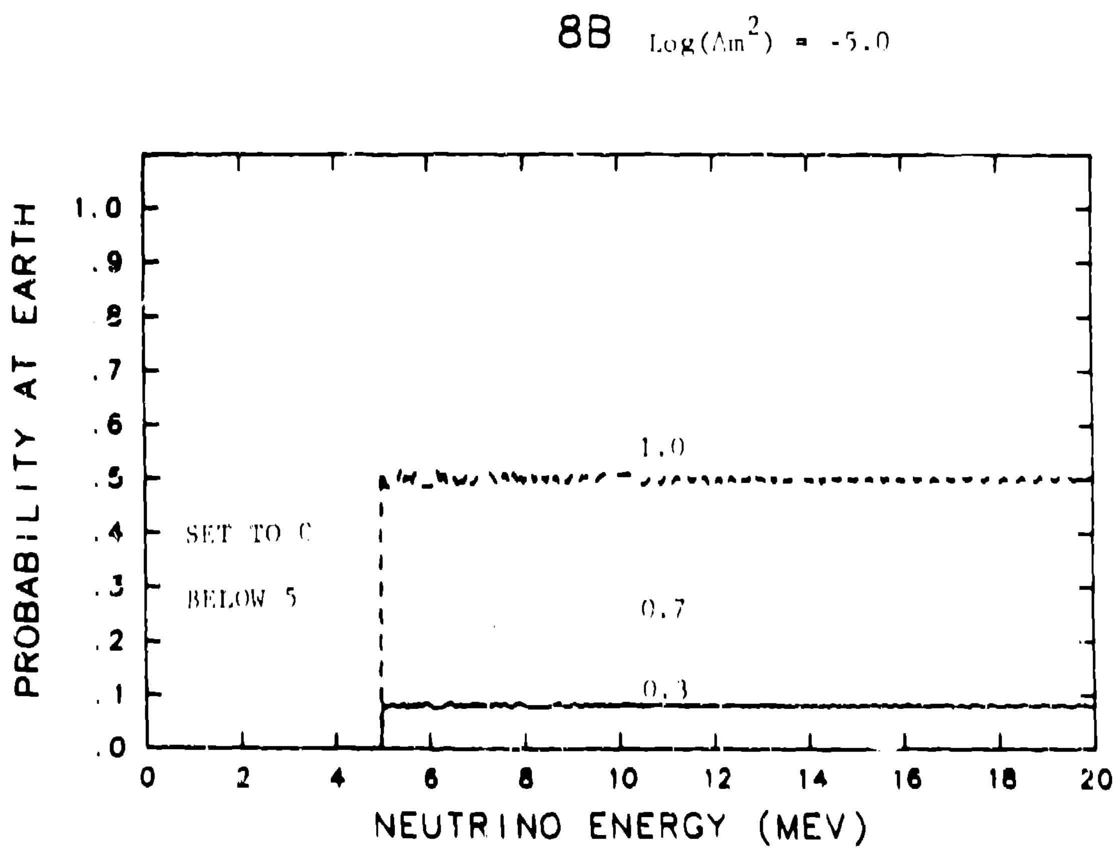
8B $Log(\Delta m^2) = -4.0$



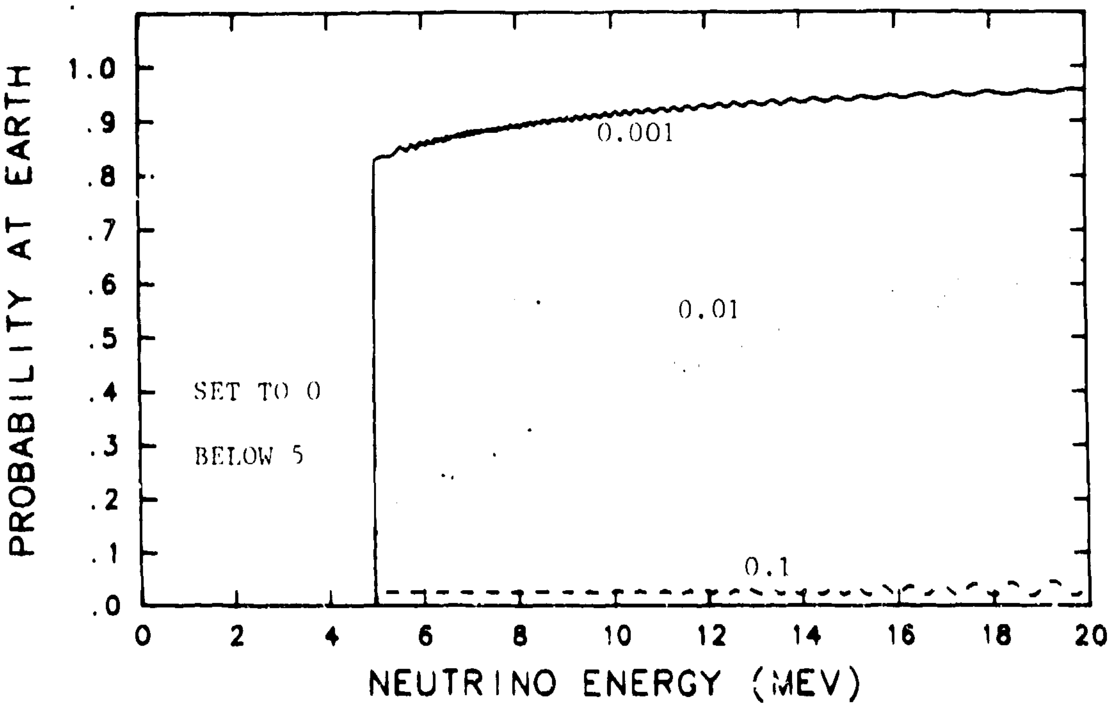

8B Log(Δm2) = -4.5
PROBABILITY AT EARTH
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
.0
0
2
4
6
8
10
12
14
16
18
20
NEUTRINO ENERGY (MEV)
0.001
SET TO 0
BELOW 5
... 0.01
--- 0.1


8B Log(Δm2) = -4.5
PROBABILITY AT EARTH
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
.0
0
2
4
6
8
10
12
14
16
18
20
NEUTRINO ENERGY (MEV,
1.0
SET TO 0
BELOW 5
0.7
0.3


8B Log(Δm²) = -5.0
PROBABILITY AT EARTH
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
.0
0.001
SET TO 0
BELOW 5
0.01
0.1
0
2
4
6
8
10
12
14
16
18
20
NEUTRINO ENERGY (MEV)


8B Log(Δm²) = -5.0
PROBABILITY AT EARTH
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
.0
1.0
SET TO 0
BELOW 5
0.7
0.3
0
2
4
6
8
10
12
14
16
18
20
NEUTRINO ENERGY (MEV)

8B $\text{Log}(\Delta m^2) = -5.5$



8B $\text{Log}(\Delta m^2) = -5.5$

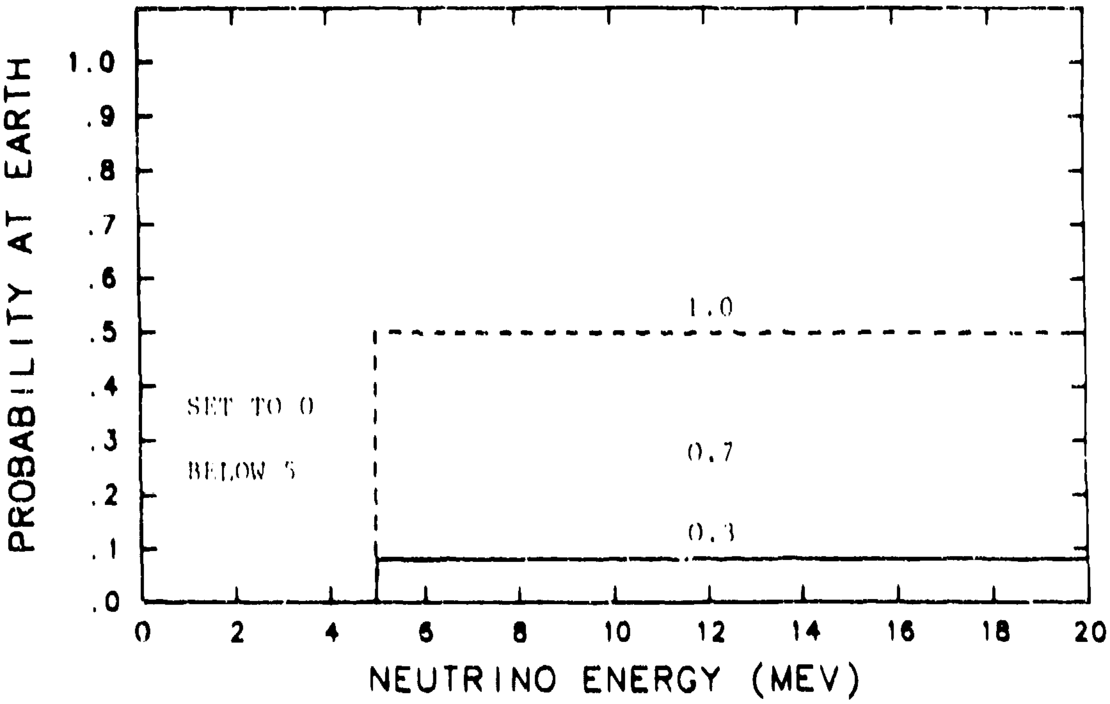
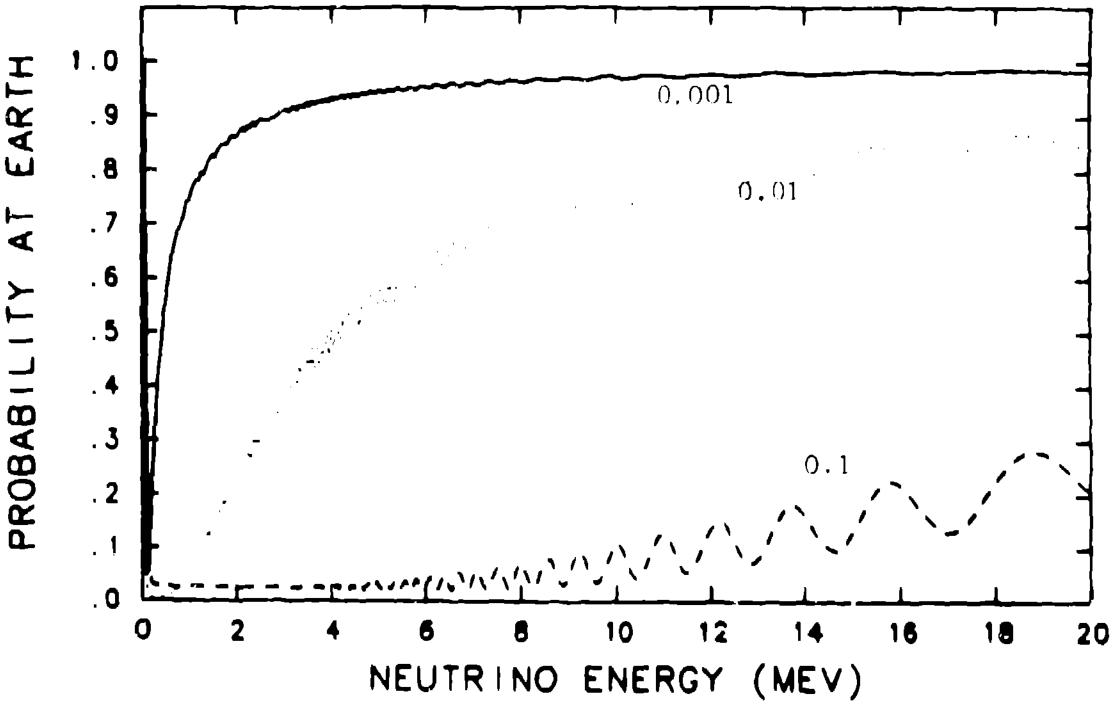


FIGURE 6

8B $Log(\Delta m^2)$ = -6.0



8B $Log(\Delta m^2)$ = -6.0

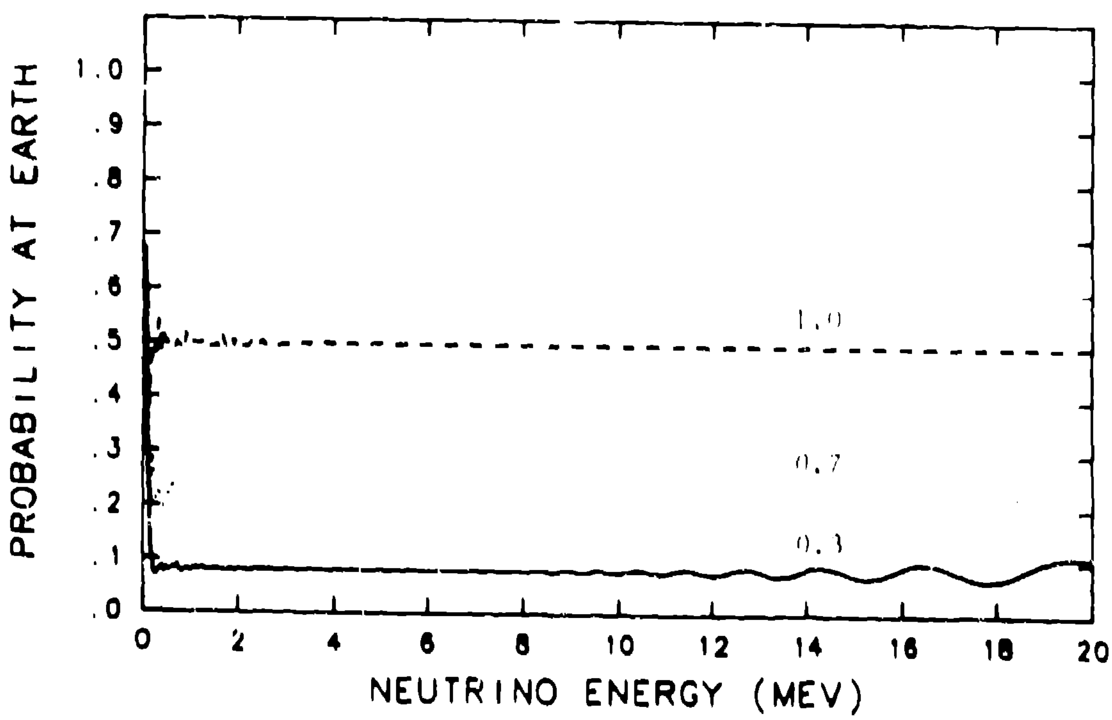
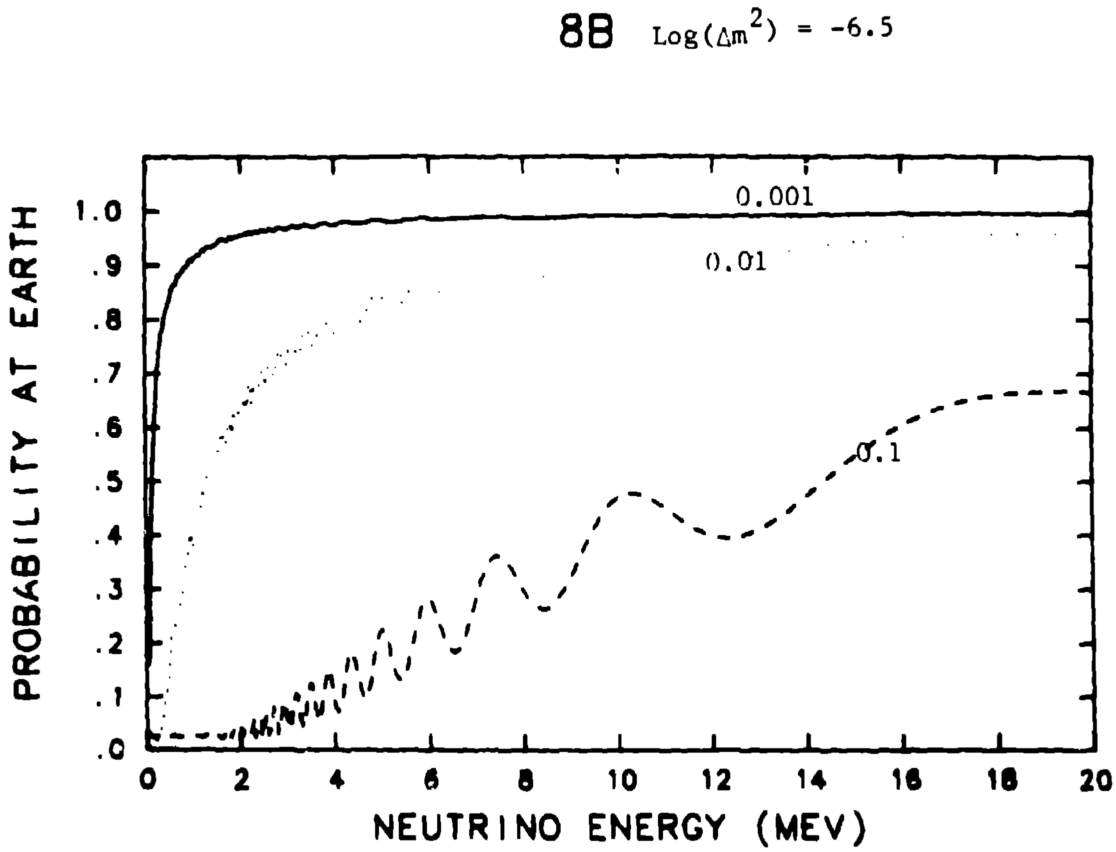
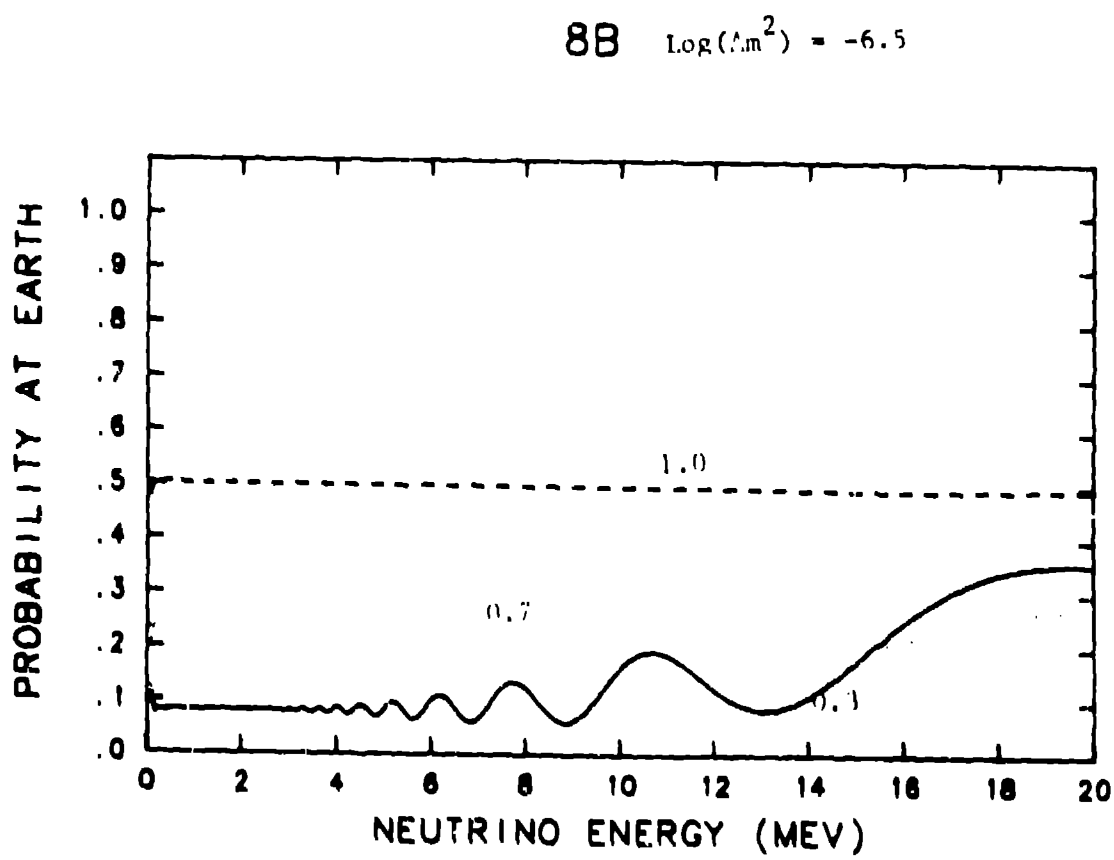
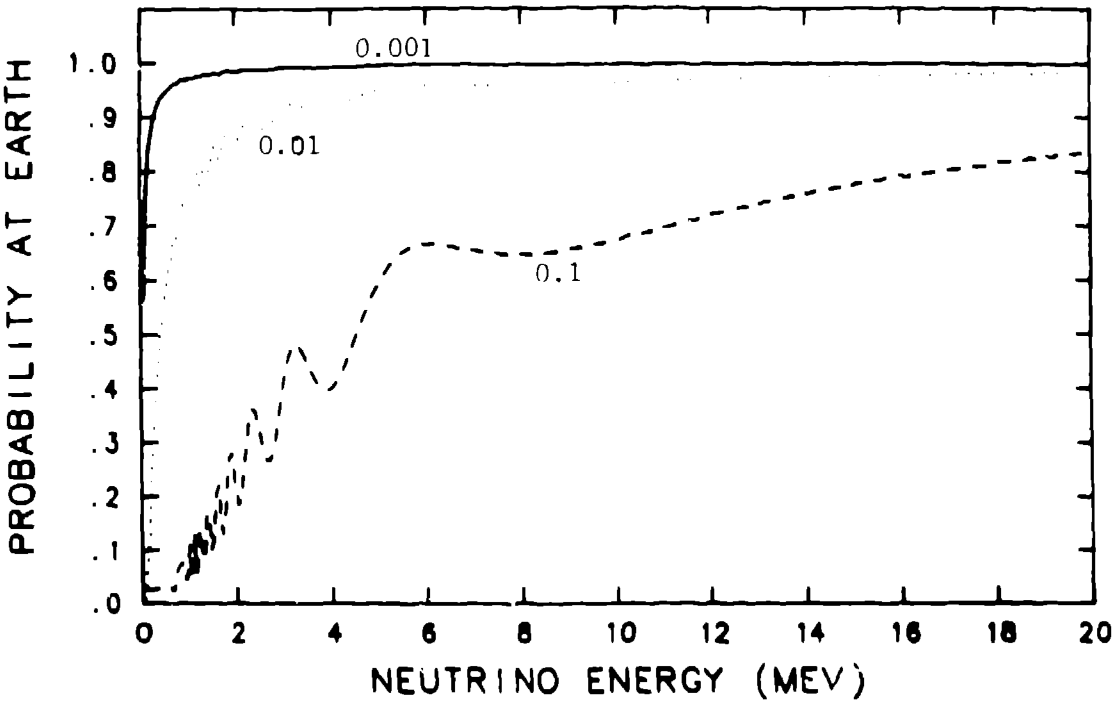


FIGURE 7

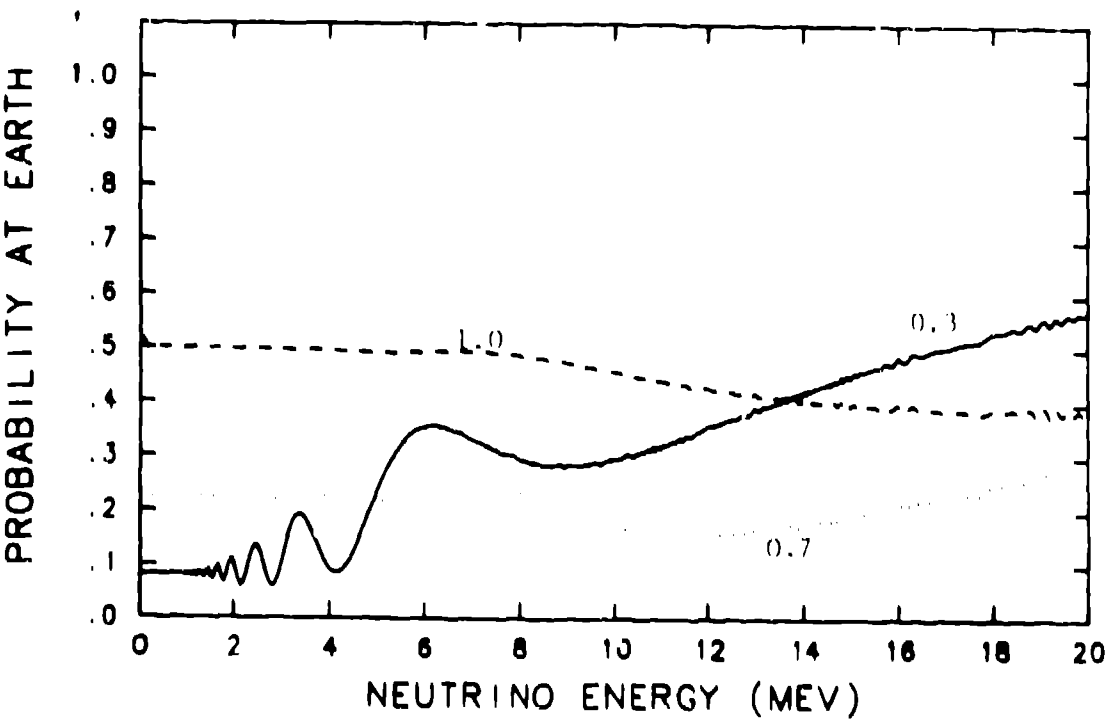

8B Log(Δm^2) = -6.5
PROBABILITY AT EARTH
NEUTRINO ENERGY (MEV)
0.001
0.01
0.1
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
.0
0
2
4
6
8
10
12
14
16
18
20


8B Log(Δm^2) = -6.5
PROBABILITY AT EARTH
NEUTRINO ENERGY (MEV)
1.0
0.7
0.3
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
.0
0
2
4
6
8
10
12
14
16
18
20

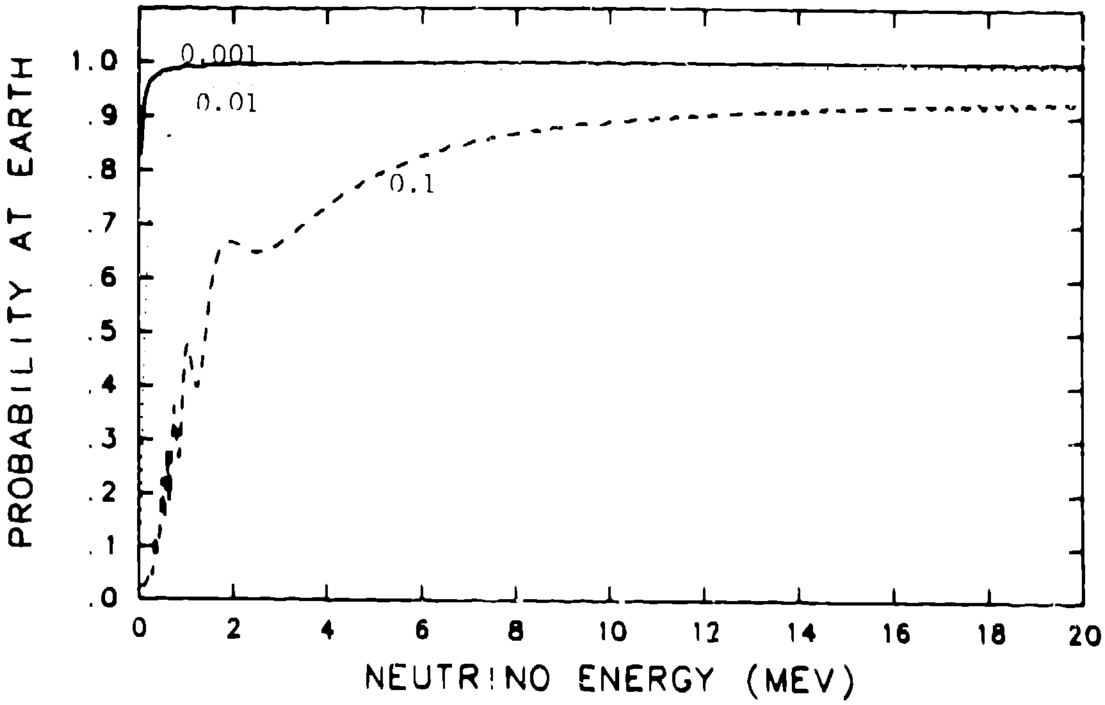
8B Log(Δm^2) = -7.0



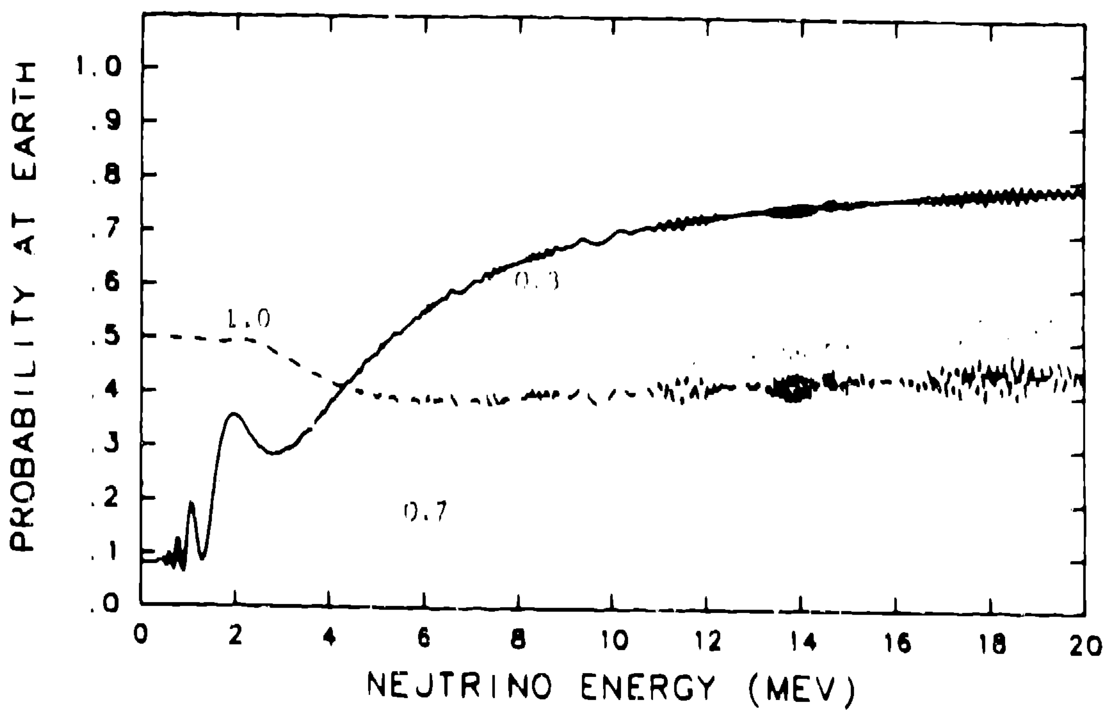
8B Log(Δm^2) = -7.0



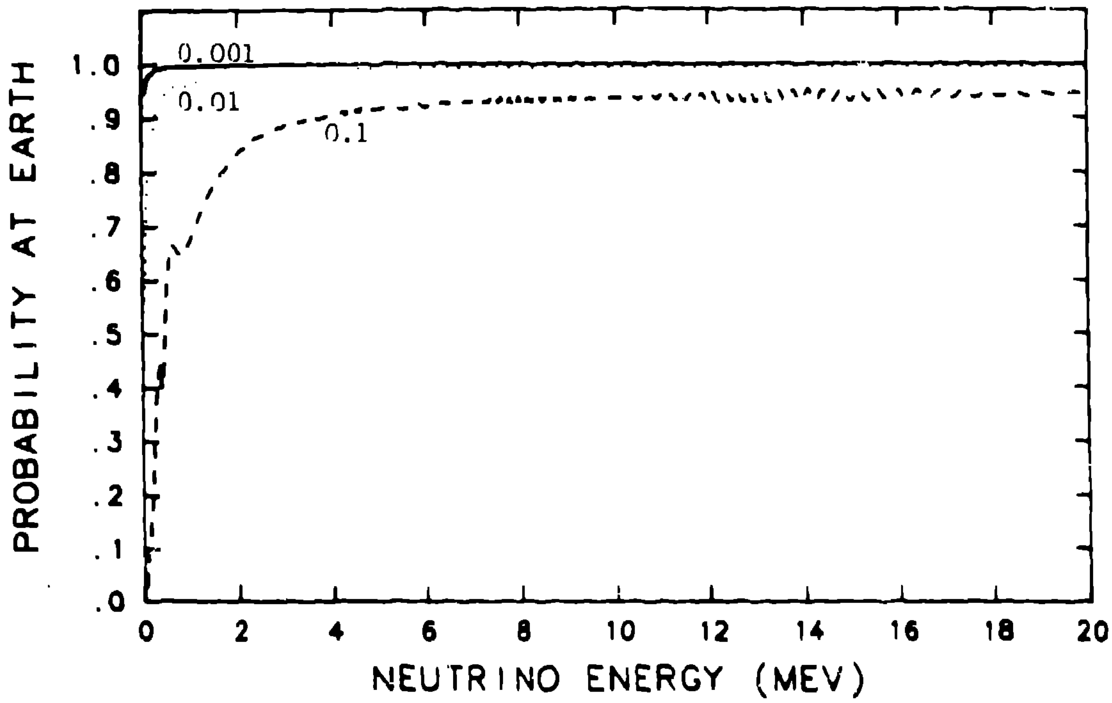
8B $\text{Log}(\Delta m^2) = -7.5$



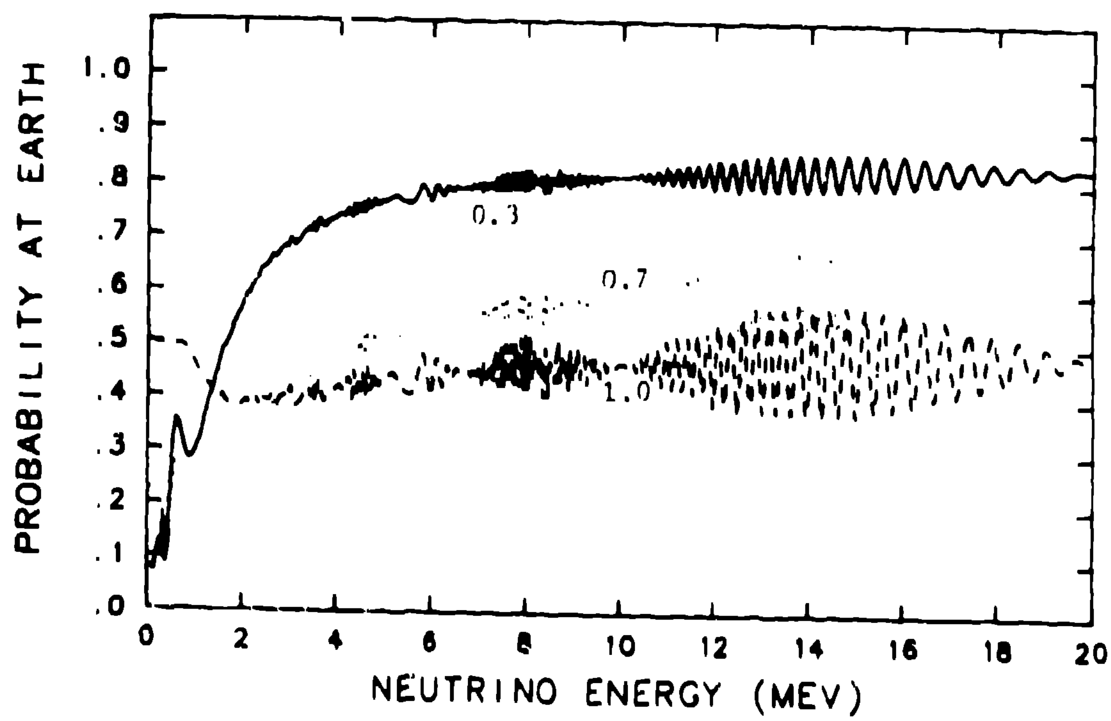
8B $\text{Log}(\Delta m^2) = -7.5$



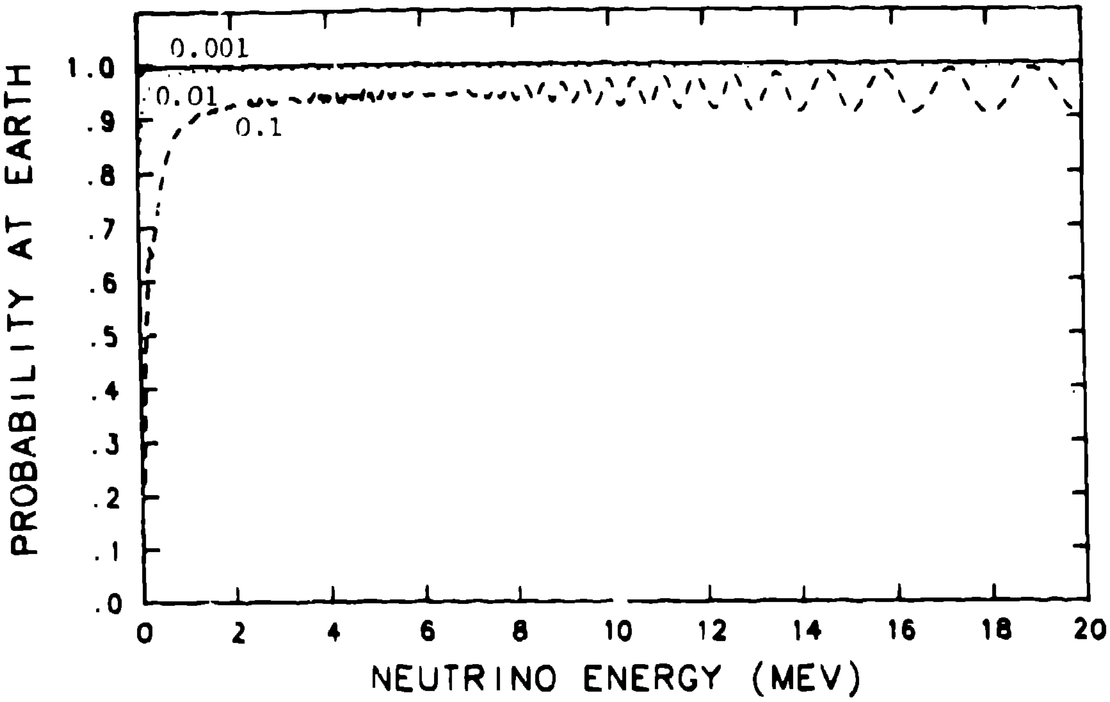
8B $Log(\Delta m^2) = -8.0$



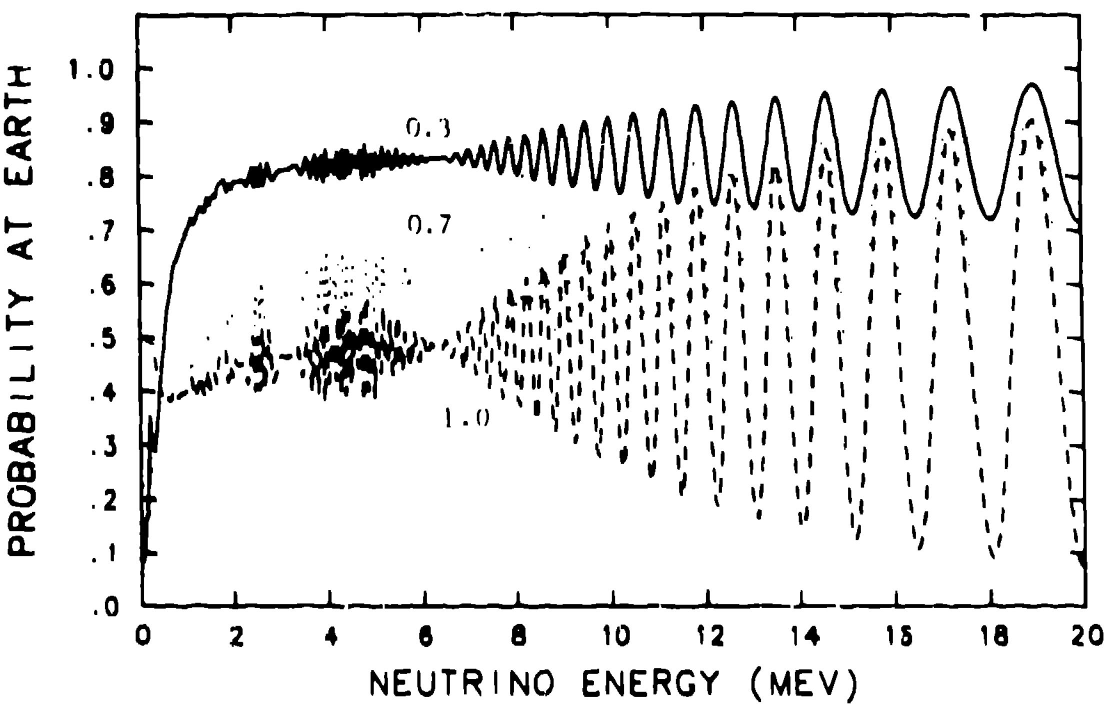
8B $Log(\Delta m^2) = -8.0$



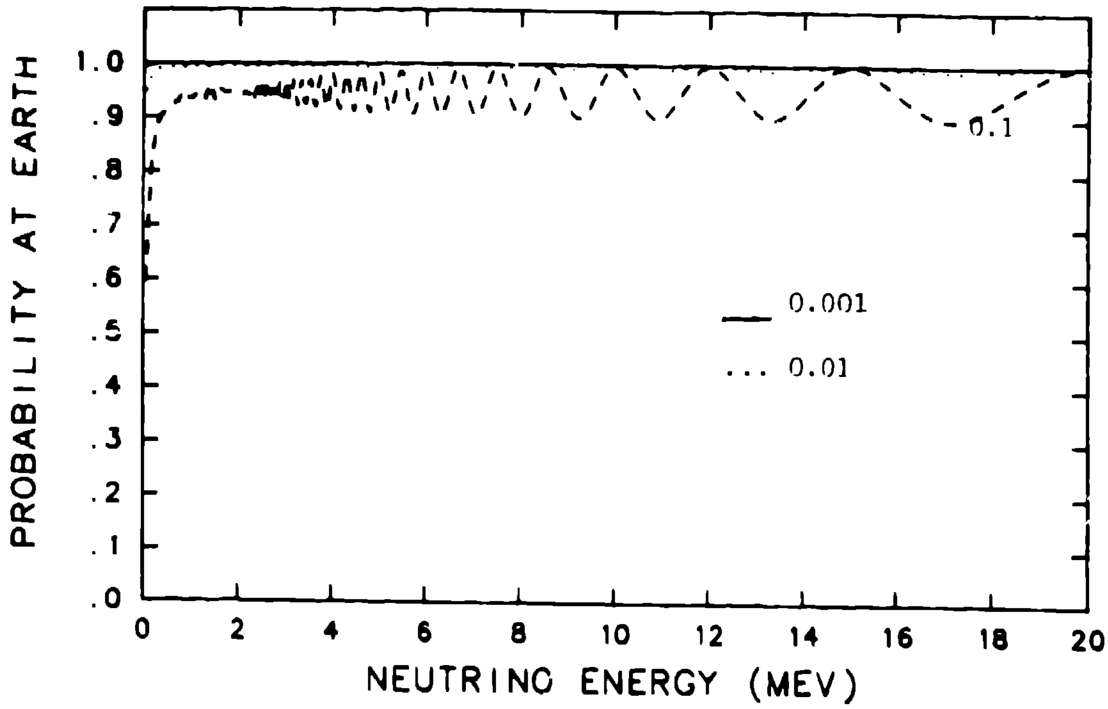
8B $Log(\Delta m^2) = -8.5$



8B $Log(\Delta m^2) = -8.5$



8B $Log(\Delta m^2) = -9.0$



8B $Log(\Delta m^2) = -9.0$

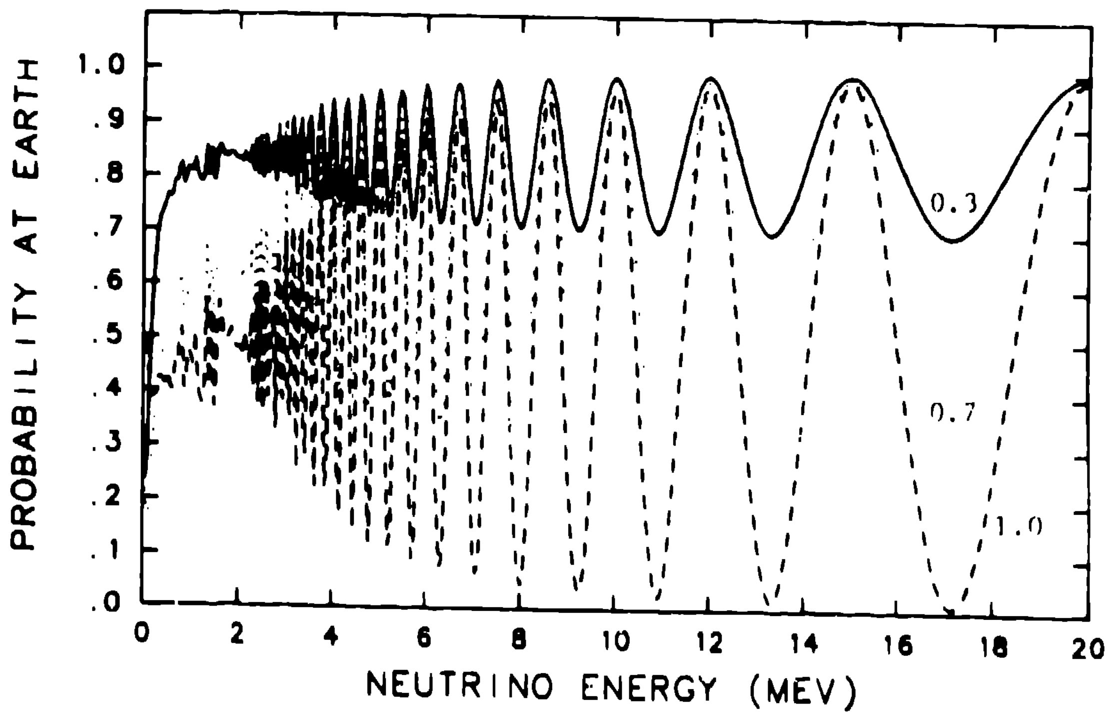
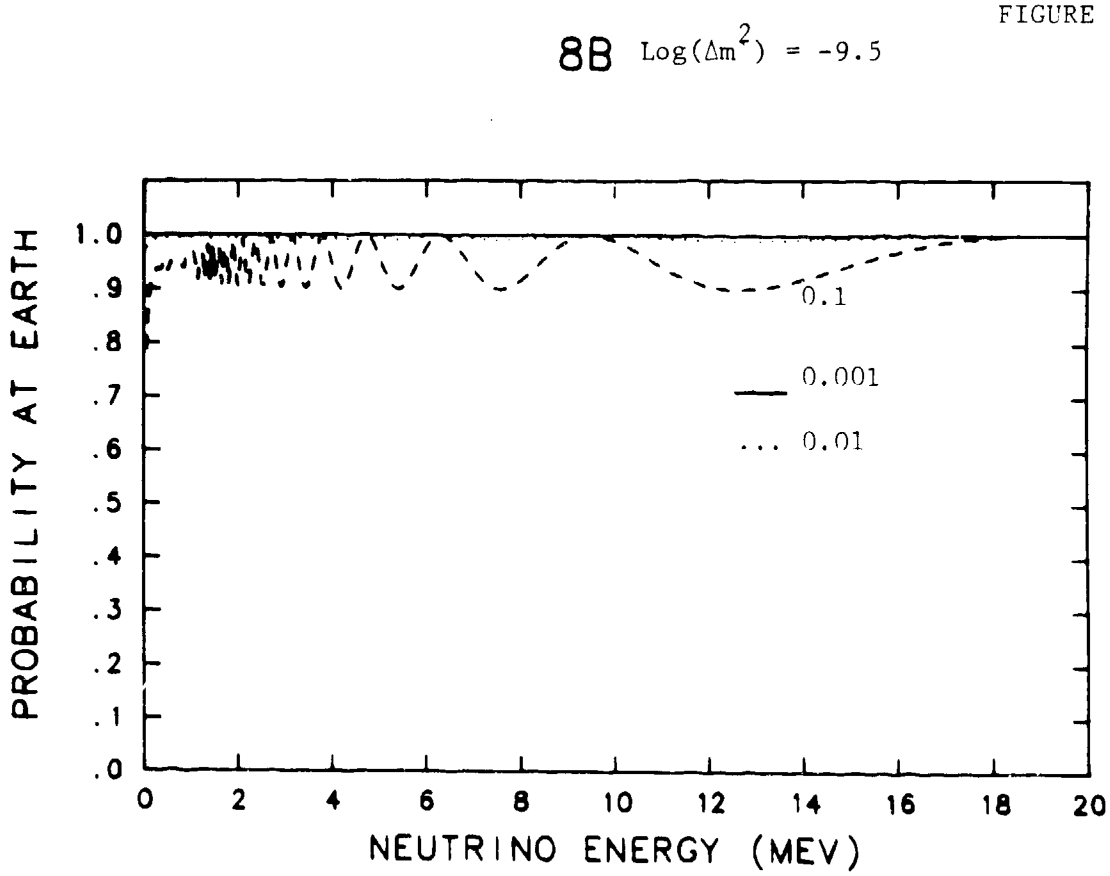
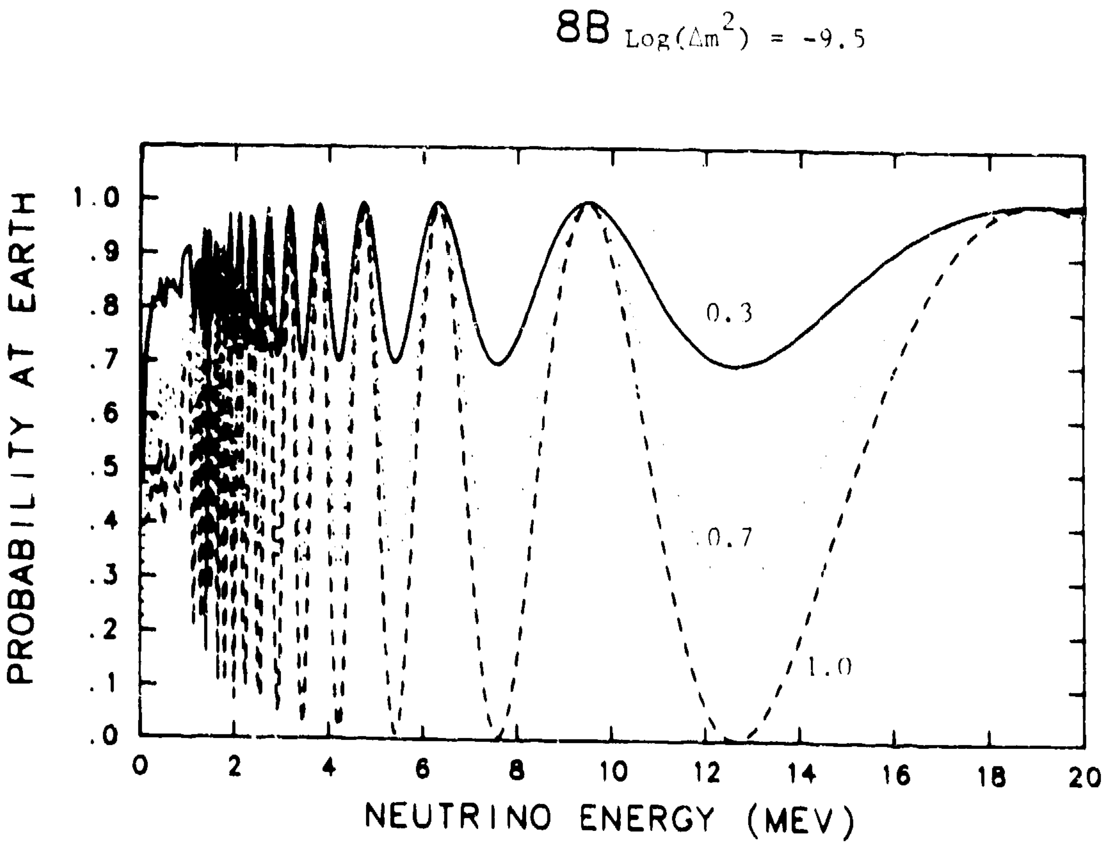


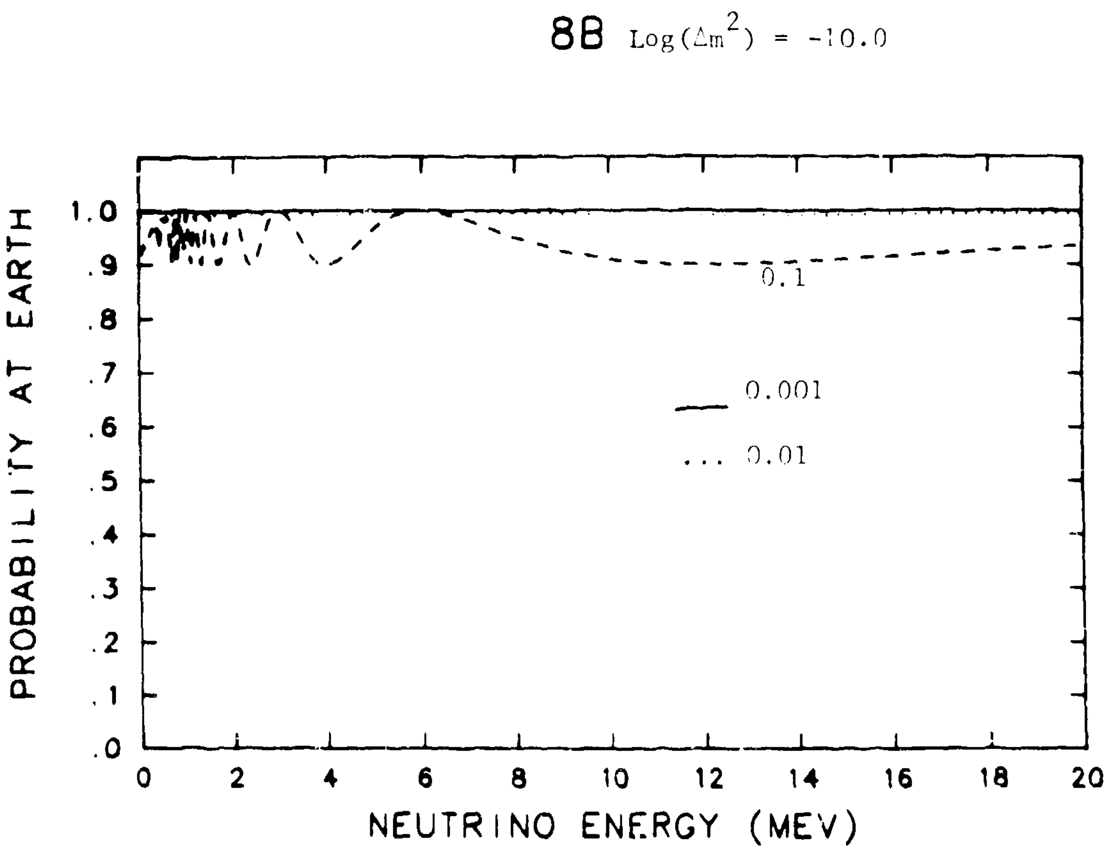
FIGURE 13



8B Log(Δm²) = -9.5
PROBABILITY AT EARTH
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
.0
0
2
4
6
8
10
12
14
16
18
20
NEUTRINO ENERGY (MEV)
0.1
0.001
0.01



8B Log(Δm²) = -9.5
PROBABILITY AT EARTH
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
.0
0
2
4
6
8
10
12
14
16
18
20
NEUTRINO ENERGY (MEV)
0.3
0.7
1.0


8B Log(Δm^2) = -10.0
PROBABILITY AT EARTH
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
.0
0
2
4
6
8
10
12
14
16
18
20
NEUTRINO ENERGY (MEV)
0.1
0.001
0.01

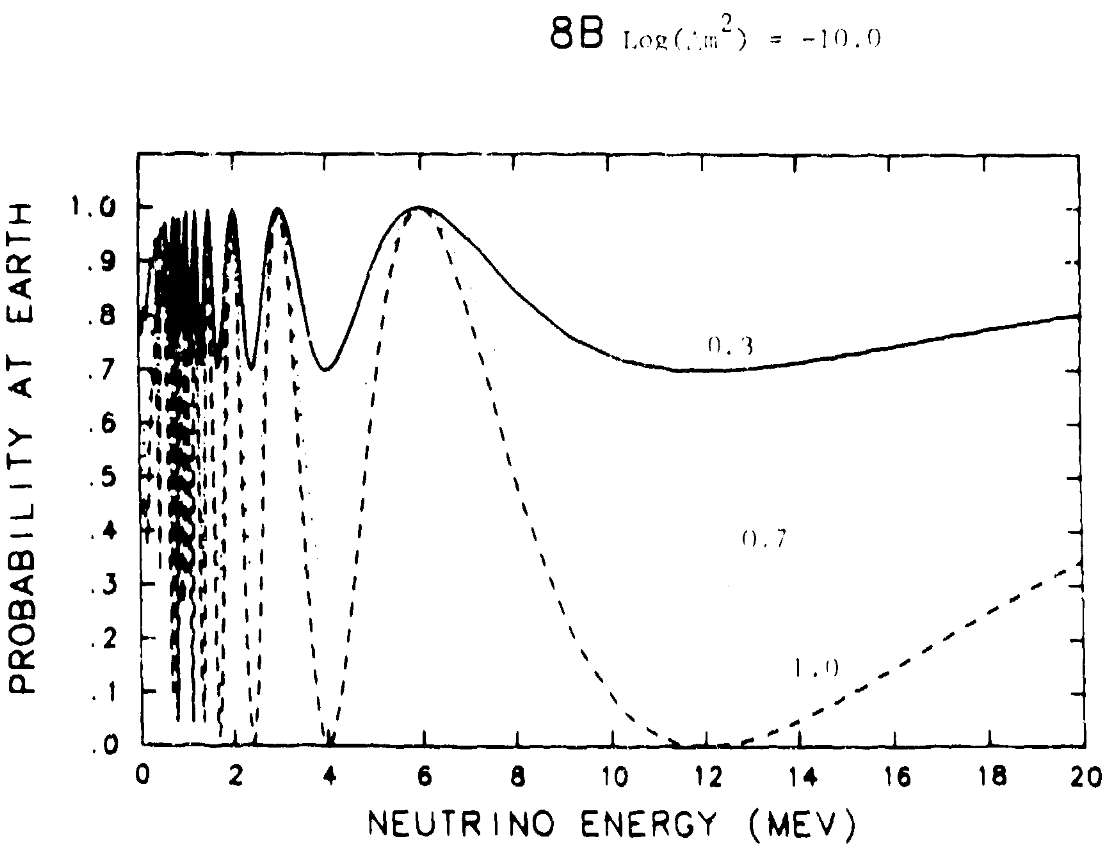

8B Log(Δm^2) = -10.0
PROBABILITY AT EARTH
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
.0
0
2
4
6
8
10
12
14
16
18
20
NEUTRINO ENERGY (MEV)
0.3
0.7
1.0

FIGURE 15

NORMALIZED 8B SPECTRUM